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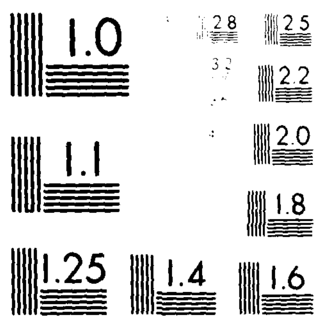

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LINOPT, A FORTRAN ROUTINE FOR SOLVING LINEAR PROGRAMMING PROBLEMS

BY JOHN W. WINGATE

RESEARCH AND TECHNOLOGY DEPARTMENT

9 OCTOBER 1981

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This report documents a FORTRAN routine LINOPT for solving linear programming problems. Upper and lower bounds on all variables are permitted, and the dual problem includes as a special case linearly-constrained minimum ℓ_1-norm problems. Basic theory, the algorithm used, input-output procedures and examples of use are included.

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FOREWORD

This report documents a FORTRAN routine LINOPT for solving linear programming problems. Upper and lower bounds on all variables are permitted, and the dual problem includes as a special case linearly-constrained *minimum* ℓ_1-norm problems. Basic theory, the algorithm used, input-output procedures and examples of use are included.

Ira M Blatstein

IRA M. BLATSTEIN
By direction

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CHAPTER 1

INTRODUCTION

REPORT ORGANIZATION

This report documents a FORTRAN subroutine (with associated subroutines) called LINOPT for solving linear programming (LP) problems. It is divided into several chapters. Following the INTRODUCTION is a chapter (PROBLEM FORMAT AND PROGRAM USE) explaining the types of problems which can be solved by LINOPT, some manipulations on them and correspondences with program notation. A chapter (SUPPORT FUNCTIONS AND DUALITY IN LINEAR PROGRAMMING) discusses some of the duality concepts behind the formulation and solution of LP problems. The next chapter, DUAL SIMPLEX METHOD gives some information about the algorithm used in the program. EXAMPLES and a LISTING follow.

PROBLEM FORMULATION

A rather abstract formulation of an LP problem in the following: Let X be a real vector space paired with M under the bilinear form (inner product) $(\mu,x) \longrightarrow \mu\cdot x$, $\mu\varepsilon M$, $x\varepsilon X$, and let Y_i, $i = 1,\ldots.,m$ be similarly paired with Λ_i, $i = 1,\ldots, m$. Given $\mu\varepsilon M$, closed convex sets C_i in Y_i, and linear transformations A_i, : $X \to Y_i$, $i = 1,\ldots,m$,

$$\begin{array}{l} \text{maximize} \quad \mu\cdot x \\ \text{subject to } A_i x \;\varepsilon\; C_i,\ i=1,\ldots,m. \end{array}$$

This problem in convex programming has the following as its dual problem:

$$\text{minimize} \sum_{i=1}^{m} \sigma C_i(\lambda_i)$$

$$\text{subject to} \sum_{i=1}^{m} A_i^{*}\lambda_i = \mu$$

where $^{\sigma}C_i$ is the support function of the set C_i — see Chapter 3 for more on support functions. For explanatory purposes it is sufficient to take each A_i equal to the identity, so that $Y_i = X$, $\Lambda_i = M$, $i = 1,\ldots,m$. Chapter 3 treats this simplified version.

The program LINOPT is set up to handle constraints of the form $\mu_i.x \;\varepsilon\; C_i$ where $\mu_i \;\varepsilon\; M$ and C_i is a nonempty closed interval. If some C_i is a bounded interval of nonzero length, the dual problem has a nonlinear objective; it is, however, convex and piecewise linear. The details are given in Chapter 2, where it is shown that the general form of the dual objective is the sum of two terms, one linear, and one a weighted ℓ_1 norm.

The algorithm used in the program is a form of the revised dual simplex algorithm modified to handle upper and lower bound constraints. The inverse matrix used is a row-basis inverse. Accordingly, the algorithm is more efficient on problems with many constraints. (On problems with fewer dependent variables than independent variables, a column-basis inverse would be smaller.) Use of a row basis has definite advantages in modifying a problem and then reoptimizing.

No new theory is involved in this program. The dual simplex algorithm can be found in standard linear programming texts.[1,2] Insisting that all variables in the primal problem have both upper and lower bounds makes it trivial to find a dual-feasible point to start the algorithm, and because of the resulting asymmetry between primal and dual problems, allows us to handle directly (via the dual problem) certain piecewise linear convex minimization problems.

The results from convex analysis used in Chapter 3 can be found in greater generalization and detail in Rockafellar's book.[3]

---

[1]Hadley, G., Linear Programming, Addison-Wesley, Reading, 1962.

[2]Simmonard, M., Linear Programming, Prentice-Hall, Englewood Cliffs, 1966.

[3]Rockafellar, R. T., Convex Analysis, Princeton University Press, Princeton, 1970.

CHAPTER 2

PROBLEM FORMAT AND PROGRAM USE

INTRODUCTION

LINOPT is programmed to solve a problem maximizing (or minimizing) a linear function subject to upper and lower bounds on linear constraint functions. (The bounds are equal for an equality constraint.) The dual to this problem has a piecewise linear objective function and linear equality constraints. Missing bounds in the primal problem correspond to sign constraints on the dual variables. Such missing bounds can be handled by introducing a penalty function for the sign constraint violations in the dual problem. An even simpler and more direct interpretation is that the missing bounds can be replaced by bounds so large in magnitude that they are effectively infinite.

PRIMAL PROBLEM

Maximize x_{k_0} subject to $\underline{b}_k \leq x_k \leq \bar{b}_k$, $k=1,\ldots, m+n$, where $k_0 \in \{1,\ldots., m+n\}$ and

$$x_{n+i} = \sum_{j=1}^{n} a_{ij}\, x_j,\ i=1,\ldots.,m.$$

The objective variable x_{k_0} can be expressed by:

$$x_{k_0} = \sum_{j=1}^{n} c_j\, x_j,$$

where

$$c_j = \begin{cases} \delta_{k_0 j} & \text{if } k_0 \leq n, \\ a_{k_0-n,j} & \text{if } k_0 > n. \end{cases}$$

The same data also define the dual problem.

DUAL PROBLEM

Minimize $\sum_{k=1}^{m+n} \max\,\{u_k\underline{b}_k,\ u_k\bar{b}_k\}$ subject to

$$u_j + \sum_{i=1}^{m} u_{n+i}\, a_{ij} = c_j,\ j=1,\ldots,n.$$

The dual objective has another form which is more likely to be recognized in an application:

$$\sum_{k=1}^{m+n} \max\{u_k\underline{b}_k,\ u_k\bar{b}_k\} = \sum_{k=1}^{m+n} \left(\frac{\bar{b}_k + \bar{b}_k}{2}\right) u_k + \sum_{k=1}^{m+n} \left(\frac{\bar{b}_k - \underline{b}_k}{2}\right) |u_k|$$

Thus the dual objective contains a linear term and a weighted ℓ_1-norm term.

The dual variable u_k can be thought of as a Lagrange multiplier for the constraint $x_k \in [\underline{b}_k, b_k]$.

MISSING BOUNDS; SIGN CONSTRAINTS

The following table (TABLE 1) shows how to prepare primal problems with missing constraints or dual problems with sign constraints. M is a very large positive number. (The default value supplied by the program is 10^{100}.) The first line of the table gives the standard two-sided constraint assumed by the program. The other lines give the modifications for unilateral and no constraints. In the modified problem u_k is always unconstrained in sign. The dual objective picks up the original linear term when the sign constraint is satisfied, and a penalty term when the sign constraint is violated.

When the program gives an optimal solution in which $x_k = \pm M$ for some k, the original problem has an unbounded solution. If it has a finite solution, the program will yield it, and it will not depend on M (unless M has been set so small that it interferes with the "legitimate" constraints). The calculations are arranged so that roundoff errors due to the disparity in magnitude between M and the original data do not propagate from iteration to iteration, and appear within an iteration only if some $x_k = \pm M$.

The objective variable x_{k_0} is also formally a constrained variable, although generally the constraint will be $-M \leq x_{k_0} \leq M$, i.e. essentially no constraint at all. Tighter bounds may at times be useful. The constraint $\underline{b}_{k_0} \leq x_k \leq M$ can be used to answer the question: Is max $x_{k_0} \geq \underline{b}_{k_0}$? If the answer to this question is negative, the constraints are inconsistent. As soon as the inconsistency is detected, the program returns to the calling program without going on to calculate the solution completely.

TABLE 1 FILLING IN MISSING BOUNDS

Original constraints on x_k	Original sign constraints on u_k	Original dual objective term	Modified constraints	Modified dual objective term
$\underline{b}_k \leq x_k \leq \bar{b}_k$	u_k unconstrained in sign	$\max\{u_k\underline{b}_k,\ u_k\bar{b}_k\}$	$\underline{b}_k \leq x_k \leq \bar{b}_k$	$\max\{u_k\underline{b}_k, u_k\bar{b}_k\}$
$x_k \leq \bar{b}_k$	$u_k \geq 0$	$u_k\bar{b}_k$	$-M \leq x_k \leq \bar{b}_k$	$\max\{u_k(-M), u_k\bar{b}_k\}$
$x_k \geq \underline{b}_k$	$u_k \leq 0$	$u_k\underline{b}_k$	$\underline{b}_k \leq x_k \leq M$	$\max\{u_k\underline{b}_k, u_kM\}$
x_k unconstrained	$u_k = 0$	0	$-M \leq x_k \leq M$	$M\lvert u_k\rvert$

PROGRAM NOTATION

Correspondence between the notation herein and the notation of LINOPT is given in Table 2.

TABLE 2 CORRESPONDING VARIABLES

This TR	LINOPT
m	M
n	N
k_0	IOBJ
x_k	X(K)
u_k	U(K)
$\underline{b}_k$	BL(K)
$\bar{b}_k$	BU(K)
a_{ij}	A(ROW(I) + COL(J))
M	BIGM

The FORTRAN variable BIGM is included in the program for the user's convenience. It supplies a default value (which can be changed) for filling in the missing bounds. (The user must fill in all bounds, since there is no provision for keeping track of missing bounds otherwise.)

CONSTRAINT COEFFICIENT STORAGE

The constraint coefficients are referenced in an unusual but flexible way. Row and column pointer arrays ROW and COL are used to index an array A. The FORTRAN standard for array storage is by columns (a_{11}, a_{21}, a_{31},..., a_{12}, a_{22}, a_{32},..., etc.) Suppose that we have a matrix A stored in an array dimensioned 10 x 20 and we wish to study a problem whose constraint coefficients form a submatrix of A, as in

$$\begin{bmatrix} y_5 \\ y_8 \end{bmatrix} = \begin{bmatrix} a_{57} & a_{53} & a_{59} \\ a_{87} & a_{83} & a_{89} \end{bmatrix} \begin{bmatrix} z_7 \\ z_3 \\ z_9 \end{bmatrix}$$

We can set $x_1 = z_7$, $x_2 = z_3$, $x_3 = z_9$, $x_4 = y_5$, $x_5 = y_8$,

ROW(1) = 5, ROW(2) = 8,
COL(1) = 60, COL(2) = 20, COL(3) = 80.

COL(J) is set to the number of elements in the array preceeding the coefficient column for X(J)— 6 x 10 for the 7th column, 2 x 10 for the 3rd and 8 x 10 for the 9th. ROW(I) then picks off the appropriate entry in the column. We can even introduce (by using the LOCF function)columns extraneous to the array storing A. Consider the modified example:

$$\begin{bmatrix} y_5 \\ y_8 \end{bmatrix} = \begin{bmatrix} a_{57} & a_{53} \\ a_{87} & a_{84} \end{bmatrix} \begin{bmatrix} z_7 \\ z_3 \end{bmatrix} + \begin{bmatrix} b_{54} \\ b_{84} \end{bmatrix} w_4$$

in which $x_3 = w_4$. COL(3) is set by

$$COL(3) = LOCF (B1,4)) - LOCF (A(1,1)).$$

The columns of B must be structured the same way as the columns of A for this procedure to work. Instead of having a coefficient matrix stored in the array in the usual way we could have it transposed, perhaps as a result of starting with a dual problem. This corresponds to storage by rows (not the FORTRAN standard). A little reflection indicates that defining ROW the way COL is defined above, and COL the way ROW is defined handles this storage arrangement. Further examples are given in the program comments.

INPUT AND OUTPUT

Input and output variables are clearly indicated in the program comments. (The program is listed in the section titled "LISTINGS".) Arrays are passed as formal parameters. Scalers are passed by using a labelled common block /XXXLP/, which must accordingly be a common block in the calling program.

ROUNDOFF CONTROL

In the program there are three input variables which can be used to control roundoff error accummulations. EPS is a tolerance used in checking constraint violations. H is also used to zero out coefficients in the tableau which have small nonzero values (typically for a CDC 60-bit machine, on the order of 10^{-14}) which ought to vanish. Finally constraint violations less than EPS are eliminated from the optimal solution before returning. For small problems EPS = 0 is generally all right.

The other roundoff - controlling variables are invert (a logical variable) and ITMAX. When INVERT is TRUE the inverse matrix corresponding to the current key K is calculated. ITMAX is a limit on the number of iterations. When this limit is reached control is returned to the calling LINOPT again with INVERT = TRUE., one may control the building of roundoff error in the inverse matrix (which otherwise is updated by column operations every iteration). (For small problems this may not be necessary.)

After obtaining a solution (or after any return from an initial call to LINOPT) INVERT can be set to FALSE for another call to LINOPT. Certain modifications to the problem data are permissible at such a time - constraints may be added, for example. These modifications are any for which the inverse matrix would be unchanged, and include the following (Note: primary indices: K(1),...,K(N); secondary K(N + 1),...,K(N + M) - see Chapter 4.)

ADDING CONSTRAINTS

M is increased, new elements to ROW are added to point to the new constraint coefficients (which, if not already defined should be stored appropriately), and new bounds added to BL and BU.

MODIFYING CONSTRAINTS

Bounds for any secondary variable can be changed. Inactive bounds for primary variables can be changed. Active bounds for primary variables can be changed, provided the corresponding solution is also changed; e.g. if X(K(1)) = *BU(K(1)) and BU(K(1)) is changed, X(K(1)) must be changed in the same way.* Constraint coefficients for dependent variables X(N + 1),..., X(N + M) which are also secondary variables can be changed; (these may include the objective variable) or such constraints can be dropped, with appropriate changes to ROW, M, BL and BU. (If the constraint corresponding to ROW(I) is dropped, the simplest way to make these changes is to set ROW(I) = ROW(M), BL(N + I) = BL(N + M), BU(N + I) = BL(N + M), and then M = M - 1, so that the indexing for X(N + M) *is changed to* X(N + I)).

Of course, when LINOPT is recalled with INVERT = TRUE, any problem changes whatsoever are permissible.

CHAPTER 3

SUPPORT FUNCTIONS AND DUALITY IN LINEAR PROGRAMMING

The support function σ_C of a convex set C in X is a convex function defined on M by:

$$\sigma_C = (\mu) = \sup_{x \in C} \mu \cdot x \qquad (1)$$

The support function of the empty set is $-\infty$ everywhere. For nonempty C, $\sigma_C(\mu) > -\infty$ and may take the value $+\infty$; in Rockafellar's terminology, it is a proper convex function[4].

Many of the formulas of convex analysis can be simplified when they are restricted to convex polyhedra and convex polyhedral functions. One such is found in Corollary 16.4.1 of Rockafellar's book[5], from part of which we can derive the following: Let $C_1, \ldots, C_m$ be closed convex polyhedra with nonempty intersection C (also a convex polyhedron). Then

$$\sigma_C(\mu) = \begin{cases} \min \sum_{i=1}^{m} \sigma_{C_i}(\mu_i) \\ \text{subject to} \sum_{i=1}^{m} \mu_i = \mu \end{cases} \qquad (2)$$

(The general version of the corollary is required if C is empty.) Rockafellar terms the operation in (2) "infimal convolution", since for m = 2, $\sigma_{C_1 \cap C_2}(\mu) = \inf_\lambda (\sigma_{C_1}(\lambda) + \sigma_{C_2}(\mu - \lambda))$ a form reminiscent of integral convolutions. Formulas (1) and (2) express $\sigma_C(\mu)$ as the common optimal value of a pair of dual convex programming problems:

Primal Problem:

Maximize $\mu \cdot x$

subject to $x \in C_i$, $i = 1, \ldots, m$.

Dual problem:

Minimize $\sum_{i=1}^{m} \sigma_{C_i}(\mu_i)$

subject to $\sum_{i=1}^{m} \mu_i = \mu$.

4. Ibid, p. 24.

5. Ibid, p. 146.

Three solution cases arise:

1. ${}^{\sigma}C\ (\mu) = -\infty$: C is empty. The dual objective is unbounded below on the dual constraint set.

2. $-\infty < {}^{\sigma}C\ (\mu) < +\infty$: C is not empty. The optimal value, viz ${}^{\sigma}C\ (\mu)$, is attained in both problems.

3. ${}^{\sigma}C\ (\mu) = +\infty$: C is not empty. The primal objective is unbounded above on the primal constraint set. The dual objective is $+\infty$ everywhere on the dual constraint set, (i.e. ${}^{\sigma}C_i\ (\mu_i) < \infty,\ i = 1,\ldots,m \Rightarrow \Sigma_{i=1}^{m}\ \mu_i \neq \mu$.

Suppose that ${}^{\sigma}C\ (\mu)$ is finite, and let x^* solve the primal, μ_i^*, $i=1,\ldots,m$, the dual. Then

$$
\begin{aligned}
{}^{\sigma}C\ (\mu) &= \mu \cdot x^* && \text{(primal optimality)} \\
&= (\Sigma_{i=1}^{m}\ \mu_i^*) \cdot x^* && \text{(dual constraint)} \\
&= \Sigma_{i=1}^{m}\ (\mu_i^* \cdot x^*) && \\
&\leq \Sigma_{i=1}\ {}^{\sigma}C_i\ (\mu_i^*) && \text{(primal constraints)} \\
&= {}^{\sigma}C\ (\mu) && \text{(dual optimality)}
\end{aligned}
$$

It follows that

$$\mu_i^* \cdot x^* = {}^{\sigma}C_i\ (\mu_i^*),\ i=1,\ldots,m, \tag{3}$$

or that μ_i^* supports C_i at x^*: μ_i^* is an outer normal to C_i at x^*. The formula is one way of expressing complementary slackness, since if $x^* \in \text{int}\ C_i$ then $\mu_i^* = 0$, while

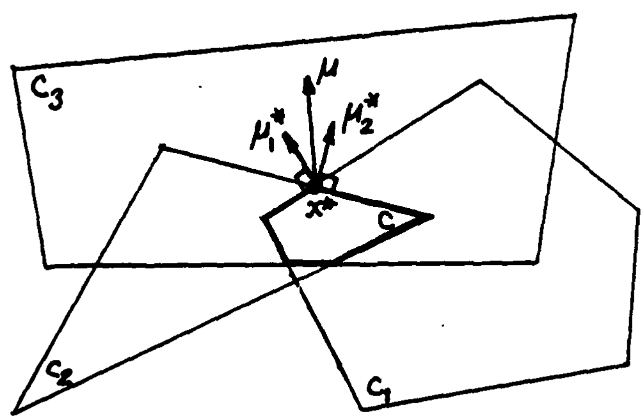

Figure 1 ILLUSTRATION OF DUAL OPTIMAL SOLUTIONS. ($\mu_3^* = 0$)

if $\mu_i^* = 0$, then the constraint $x \in C_i$ can be dropped without altering the solution. The set of constraints for which $\mu_i^* \neq 0$ are active (binding) at the optimum. (See Figure 1). There may be multiple solutions, each with a different set of active constraints.

The usefulness of (2) hinges on picking the constraint sets C_i, $i=1,\ldots,m$, to be simple enough to permit easy evaluation of their support functions. Any convex polyhedron can be expressed as the intersection of half-spaces, and any half-space can be defined by a linear inequality. Let $H := \{x: \mu\cdot x \leq b\}$, where $\mu \neq 0$. Then

$$\sigma_H(\nu) = \begin{cases} ub \text{ if } \nu = u\mu \text{ and } u \geq 0, \\ +\infty \text{ otherwise.} \end{cases}$$

In terms entirely of hyperplane constraints the primal and dual problems become Primal problem:

Maximize $\mu\cdot x$

subject to $\mu_i \cdot x \leq b_i$, $i=1,\ldots,m$

Dual problem:

Minimize $\sum_{i=1}^{m} u_i b_i$

subject to $\sum_{i=1}^{m} u_i \mu_i = \mu$,

$u_i \geq 0$, $i=1,\ldots,m$.

The sign constraints on u_i avoid infinite values of the dual objective and keep it linear so that both problems consist of optimizing linear functionals subject to linear constraints. Alternatively we could omit the sign constraints and keep the formulation in terms of support functions. This pair of problems also illustrates the more general duality relationship cited in the INTRODUCTION. Let A_i: $X\to R$ be defined by $A_i X := \mu_i \cdot X$. Then A_i^*: $R\to M$ and $A_i^* u = u\mu_i$. Moreover,

$$\sigma_{(-\infty,b_i]}(u) = \begin{cases} ub_i \text{ if } u \geq 0, \\ +\infty \text{ if } u \leq 0. \end{cases}$$

Thus the problems are expressible as:

Primal problem:

Maximize $\mu\cdot x$

subject to $A_i\, x \in (-\infty, b_i]$, $i=1,\ldots,m$

Dual problem:

Minimize $\sum_{i=1}^{m} \sigma_{(-\infty,b_i]}(u_i)$

subject to $\sum_{i=1}^{m} A_i^* u_i = \mu$

We may have two-sided constraints such as

$$\underline{b} \leq \mu \cdot x \leq \bar{b}$$

(where $\underline{b} \leq \bar{b}$) defining a set S in X, which can be replaced by the pair of constraints

$$\begin{cases} \mu \cdot x \leq \bar{b}, \\ -\mu \cdot x \leq -\underline{b}. \end{cases}$$

since

$$\sigma_S(\nu) = \begin{cases} \max\ \{u\underline{b},\ u\bar{b}\}\ \text{if}\ \nu = u\mu, \\ +\infty\ \text{otherwise}, \end{cases}$$

and $\max\ \{u\underline{b},\ u\bar{b}\}$ is nonlinear in u (unless $\underline{b} = \bar{b}$), the dual of a problem with two-sided constraints is nonlinear. Of course it is easy to relate the two-sided and one-sided versions by using (2). Thus if u is the dual variable for the constraint $\underline{b} \leq \mu \cdot x \leq \bar{b}$, u^+ for $\mu \cdot x \leq b$ and u^- for $-\mu \cdot x \leq -\bar{b}$,

then

$$u = u^+ - u^- \tag{4}$$

while if $\underline{b} \neq \bar{b}$ either u^+ or u^- vanishes (at the solution), so that

$$u^+ = \max\ \{u,0\},\ -u^- = \min\ \{u,0\}. \tag{5}$$

When $\underline{b} = \bar{b}$, only the difference $u = u^+ - u^-$ is determined. An alternate viewpoint in this case is that u is the dual variable for the linear equality constraint $\mu \cdot x = \bar{b}$.

The pivoting operations of the dual simplex method can be thought of as substituting one hyperplane bounding a half-space for another, and consequently are better suited for the formulation in terms of one-sided constraints. The relations (4) and (5) and some sign bookkeeping then make it easy to apply the method to two-sided constraints. Explanations of the method without the sign manipulations are more transparent. Accordingly in the next section only one-sided constraints are considered.

CHAPTER 4

DUAL SIMPLEX METHOD

In the previous sect'on no particular coordinates were used on X. Most LP problems encountered are expressed in terms of coordinates with respect to some particular basis for X, the coordinates then forming a set of independent variables. (as indicated previously, LINOPT assumes such a formulation.) Thus, with one-sided constraints, we get a pair of problems like the following, in which we assume that the n independent variables x_j, $j \varepsilon J$ are included in the m+n constrained variables x_k, $k \varepsilon K$; i.e. $J \subset K$.

Primal problem:

$$\text{Maximize } x_0 := \Sigma_{j \varepsilon J}\, c_j\, x_j$$

$$\text{subject to } x_k := \Sigma_{j \varepsilon J}\, a_{kj}\, x_j \leq b_k,\ k \varepsilon K.$$

(Note that $a_{kj} = \delta_{kj}$ for $k \varepsilon J$.)

Dual problem:

$$\text{Minimize } \Sigma_{k \varepsilon K}\, u_k\, b_k$$

$$\text{subject to } \Sigma_{k \varepsilon K}\, u_k\, a_{kj} = c_j,\ j \varepsilon J$$

$$u_k \geq 0,\ k \varepsilon K.$$

Note that the n equations relating the dual variables u_k, $k \varepsilon K$ can be written explicitly for u_j:

$$\Sigma_{i \varepsilon K \sim J}\, u_i\, a_{ij} + u_j = c_j$$

Thus in the dual problem, u_i, $i \varepsilon K \sim J$, are independent and u_j, $j \varepsilon J$, are dependent. Given some other subset J' of K for which x_j, $j \varepsilon J'$, are linearly independent, we can transform the constraint relations so that x_j, $j \varepsilon J'$ are the independent variables through which the primal problem is phrased. Given such as index set J' we can define a corresponding <u>basic solution</u>. For J the definition of a basic solution is obtained by setting the independent variables x_j, $j \varepsilon J$, and u_i, $i \varepsilon K \sim J$, to their bounds and satisfying the constraint relations among the variables; $\bar{x}_k$ and $\bar{u}_k$ are the values of x_k, u_k at the basic solution.

$\bar{x}_j = b_j,\ j \varepsilon J$	(primary primal variables)
$\bar{x}_i = \Sigma_{j \varepsilon J}\, a_{ij}\, b_j,\ i \varepsilon K \sim J$	(secondary primal variables)
$\bar{u}_j = c_j,\ j \varepsilon J$	(secondary dual variables)
$\bar{u}_i = 0,\ i \varepsilon K \sim J$	(primary dual variables)

The terms <u>primary</u> and <u>secondary</u> have been introduced instead of <u>independent</u> and <u>dependent</u> because one may wish to refer to the primary variables of the problem as initially formulated as the independent variables. The split between primary and secondary depends on the set J and changes with it. The secondary primal indices are usually called basic indices in linear programming texts, since they correspond to the indices for a column basis for the constraint matrix. This terminology is a little inappropriate here, since LINOPT makes use of a row basis corresponding to the complementary set of indices - the dual basic indices in the usual description. The use of "basic" in this sense is avoided here to prevent confusion.

Furthermore, in a problem with two-sided constraints the basic indices refer to the indexing of the equivalent one-sided problem, not the indexing of the two-sided problems, so that the basic variables for J would be x_k and $-x_k$ for $k\varepsilon K{\sim}J$ and either x_j or $-x_j$ (but not both) for $j\varepsilon J$. Alternatively we may retain the "primary/secondary" notation and supplement it with some way of indicating whether a primary variable is at its upper or its lower bound. (The program simply checks the solution value against the bounding values.

In a basic solution the primary variables satisfy the constraints placed on them if all primal variables satisfy the constraints, the basic solution and J are primal-feasible. If the dual constraints are satisfied, the basic solution is dual-feasible. A basic solution which is both primal- and dual-feasible is optimal. At a basic solution both primal and dual objective variables have the value $\Sigma_{j\varepsilon J}\, c_j b_j$. The criterion for dual feasibility is simply that $c_j \geq 0$, $j\varepsilon J$.

The transformation of the constraint coefficients accompanying a change from one set of primary variables to another can be performed explicitly when needed, or it can be expressed in terms of a nonsingular matrix relating the variables.

There are two ways of doing this. Let x_j, $j\varepsilon J$ and $x_{j'}$, $j'\,\varepsilon J'$ be two sets of primary primal variables. Set $I = K{\sim}J$, $I' = K{\sim}J'$, and let x_J be a column vector whose entries are x_j, $j\varepsilon J$, etc. Using matrix notation the two ways can be described as follows:

1. Solution for $x_{I'}$

$$x_I = Ax_J \qquad \text{(A is m x n)}$$

Rearrange columns to give:

$$B\, x_{I'} = R\, x_{J'}$$

with B a nonsingular m x m submatrix of [I -A]. Then $x_{I'} = B^{-1}R\, x_{J'}$.

The columns making up B are a basis for the space spanned by the columns of [I -A]
Applied to the dual:

$$U_I A + U_J = C$$
$$U_{I'}S + U_{J'}D = C$$
$$U_{I'}\, SD^{-1} + U_{J'} = \bar{C}, \text{ where } \bar{C} = C\, D^{-1}.$$

The rows of D form a basis for the space spanned by the rows of $\begin{bmatrix} A \\ I \end{bmatrix}$.

2. Solution for x_J and substitution:

$$\begin{bmatrix} X_I \\ X_J \end{bmatrix} = \begin{bmatrix} A \\ I \end{bmatrix} \quad X_J$$

Rearrange rows to give:

$$\begin{bmatrix} X_{I'} \\ X_{J'} \end{bmatrix} = \begin{bmatrix} S \\ D \end{bmatrix} \quad X_J = \begin{bmatrix} SD^{-1} \\ I \end{bmatrix} \quad X_{J'}$$

(S and D are the same as above. $B^{-1}R = SD^{-1}$.)

For the dual:

$$[U_I \; U_J] = U_I[I \quad -A] + [0 \quad C]$$
$$[U_{I'} \; U_{J'}] = U_I[B \quad -R] + [C_{I'} \; C_{J'}]$$
$$[U_{I'} \; U_{J'}] = U_{I'}[I \quad -B^{-1}R] + [0 \quad \overline{C}]$$

where $\overline{C} = C_{J'} + C_{I'} B^{-1}R$ and is the same as before. The inverse matrix B^{-1} is the product of elementary row operations; D^{-1} is the product of elementary column operations. Either one may be used to keep track of changes. LINOPT uses D^{-1} and generates coefficients and solutions from the original constraint coefficients by:

$$\begin{bmatrix} X_I \\ X_J \end{bmatrix} = \begin{bmatrix} A \\ I \end{bmatrix} \quad D^{-1} X_{J'}$$

When a basic solution is changed, D^{-1} is updated by column operations. (We have ignored x_0: assumme that $0 \epsilon K$ so that c is a row of $\begin{bmatrix} A \\ I \end{bmatrix}$)

The dual simplex algorithm works with dual-feasible basic solutions. Given a set J defining primary primal variables, J is altered by adding an index not in J and dropping an index in J: a secondary variable replaces a primary variable. The resulting changes in the constraint coefficients can be accomplished by Gauss-Jordan pivoting. The indices entering and leaving J are chosen in such a way that dual-feasibility is maintained and the objective function value does not increase. Proof of convergence can be found in any linear programming text.[6] The procedure is:

1. Identify violated constraints: $I' = \{ i \epsilon K \sim J: \; X_i > b_i \}$.
2. If I' is empty, stop: basic solution is optimal.
3. Pick (by some heuristic) $k \epsilon I'$.
4. Identify constraints which can be dropped without being violated when x_k is set to b_k: $J' := \{ j \epsilon J: \; a_{kj} > o \}$
5. If J' is empty, stop: constraints are inconsistent.

[6] For example, Hadley, op. cit, or Simmonard, op. cit.

6. Identify subset of J' corresponding to constraints for which dual feasibility is maintained when dropped: $J'' = \{j\varepsilon J': c_j/a_{kj} = \min_{j\varepsilon J} (cj/a_{kj})\}$.
7. Pick (while applying anticycling criterion, if desired) $\ell \varepsilon J''$.
8. Update solution: $J: = J \cup \{k\} \sim \{\ell\}$. Update inverse D^{-1} by column operations. Calculate new basic solution.
9. Go to 1.

The coefficients a_{kj} and c_j are those corresponding to the current index set J, not the original one in terms of which the problem is phrased, and are calculated by post-multiplying an original constraint matrix row (or objective row) by D^{-1}.

In the program the heuristic used in step 3 is to pick the most violated constraint. In step 7 a tie for ℓ is broken randomly, a procedure which prevents cycling almost surely.

CHAPTER 5

EXAMPLES

TEST PROGRAM

A small test program to run the following examples is listed in Figure 2. The lines between the call to LINOPT and the call to TABLO merely do some cosmetic surgery on the output, replacing quantities near M in magnitude (actually those $\geq\sqrt{M}$) by $\pm$ R (machine infinities). Three examples are given, with NAMELIST inputs and the outputs from the program. Note that in all three, ROW and COL are defined to correspond to storage by rows, and the columns of the array A contain the rows of the constraint coefficient matrix.

EXAMPLE 1

This example is essentially the problem discussed in Section 1-3 of Hadley[7], with slightly modified coefficients.

$$\text{Maximize } x_8: \quad = \quad 5.0\, x_1 + 7.6\, x_2 + 8.0\, x_3 + 4.0\, x_4$$

$$\text{subject to} \qquad x_1 \geq 0,\; x_2 \geq 0,\; x_3 \geq 0,\; x_4 \geq 0,$$

$$x_5: \quad = \quad 1.5\, x_1 + 1.2\, x_2 + 2.4\, x_3 + 1.2\, x_4 \leq 2100,$$

$$x_6: \quad = \quad 1.0\, x_1 + 4.5\, x_2 + 1.0\, x_3 + 3.0\, x_4 \leq 8000,$$

$$x_7: \quad = \quad 1.5\, x_1 + 3.0\, x_2 + 3.6\, x_3 + 1.0\, x_4 \leq 5000.$$

[7] Op. cit

NAMELIST INPUT:

```
$IN
IOBJ = 8
M = 4,
N = 4,
MIN = .FALSE.,
INVERT = .TRUE.,
ITMAX = 1000,
EPS = 0.,
ROW = 0, 10, 20, 30,
COL = 1, 2, 3, 4,
A(1, 1) = 1.5, 1.2, 2.4, 1.2,
A(1, 2) = 1.0, 4.5, 1.0, 3.0,
A(1, 3) = 1.5, 3.0, 3.6, 1.0,
A(1, 4) = 5.0, 7.6, 8.0, 4.0,
BL = 4*0., 4*-1.E100,
BU = 4*1.E100, 2100., 8000., 5000., 1.E100,
K = 1, 2, 3, 4, 5, 6, 7, 8,
X = 4*0.,
$END
```

(Since the objective coefficients c_j are all nonzero for the initial tableau, it is not really necessary to preset x_1, x_2, x_3 and x_4, as Example 2 will show).

Output:

				TABLEAU			
I	BL(1)	x(I)	BU(I)	T(I, 1)	T(I, 3)	T(I, 5)	T(I, 7)
1	0.000	0.000	R	1.000	0.000	0.000	0.000
2	0.000	1625.000	R	-.125	-.800	-.417	.500
3	0.000	0.000	R	0.000	1.000	0.000	0.000
4	0.000	125.000	R	-1.125	-1.200	1.250	-.500
5	-R	2100.000	2100.000	0.000	.000	1.000	-.000
6	-R	7687.500	8000.000	-2.938	-6.200	1.875	.750
7	-R	5000.000	5000.000	-.000	.000	.000	1.000
8	-R	12850.000	R	-.450	-2.880	1.833	1.800

The objective variable, x(8), is to be maximized. ITER = 10, IERR = 0.

The tableau gives information about the primary and secondary variables at the final iteration. The primary variables are x_1, x_3, x_5 and x_7. The rows of the tableau give the coefficients of the variables expressed in terms of the primary variables. Thus, $x_4 = -1.125\ x_1\ -1.2\ x_3 + 1.125\ x_5\ -.5\ x_7$.
The dual variables are not printed explicitly but the nonzero ones can be obtained from the objective row: $u_1 = -.450$, $u_3 = -2.880$, $u_5 = 1.833$, $u_7 = 1.800$. (For a minimization problem, these should be negated.)

EXAMPLE 2

This problem also comes from Hadley[8]. It is his Problem 8-5.

Minimize x_4: $= 3x_1 \; -2x_2 + 4x_3$
subject to $x_1 \geq 0,\; x_2 \geq 0,\; x_3 \geq 0,$

x_5: $= 3x_1 + 5x_2 + 4x_3 \geq 7,$

x_6: $= 6x_1 + x_2 + 3x_3 \geq 4,$

x_7: $= 7x_1 - 2x_2 - x_3 \leq 10,$

x_8: $= x_1 - 2x_2 + 5x_3 \geq 3,$

x_9: $= 4x_1 + 7x_2 - 2x_3 \geq 2.$

Input:

```
   $IN
   IOBJ = 4,
   M = 6,
   N = 3,
   MIN = . TRUE.,
   INVERT = . TRUE.,
   ITMAX = 1000,
   EPS = 0.,
   ROW = 0, 10, 20, 30, 40, 50,
   COL = 1, 2, 3,
   A(1, 1) = 3., -2., 4.,
   A(1, 2) = 3., 5., 4.,
   A(1, 3) = 6., 1., 3.,
   A(1, 4) = 7., -2., -1.,
   A(1, 5) = 1., -2., 5.,
   A(1, 6) = 4., 7., -2.,
   BL = 3*0., -1.E100, 7., 4., -1.E100, 3., 2.,
   BU = 6*1.E100, 10., 2*1.E100,
   K = 1,2,3,4,5,6,7,8,9,
$END
```

Output:

TABLEAU

I	BL(I)	X(I)	BU(I)	T(I, 1)	T(I, 5)	T(I, 8)
1	0.000	0.000	R	1.000	1.000	0.000
2	0.000	R	R	-.333	.152	-.121
3	0.000	R	R	-.333	.061	.152
4	-R	-R	R	2.333	-.061	.848
5	7.000	R	R	-.000	1.000	.000
6	4.000	R	R	4.667	.333	.333
7	-R	-R	10.000	8.000	-.364	.091
8	3.000	3.000	R	.000	0.000	1.000
9	2.000	R	R	2.333	.939	-1.152

THE OBJECTIVE VARIABLE, X(4), IS TO BE MINIMIZED.
ITER = 3 IERR = 0

8. Ibid., p. 267.

This problem has an unbounded solution: $X_5 = +\infty$ and the objective value is $-\infty$.

EXAMPLE 3

This example illustrates the solution of a dual problem.

$$\text{Minimize} \sum_{k=3}^{9} |u_k|$$

$$\text{subject to} \sum_{k=3}^{9} u_k = 1,$$

$$\sum_{k=3}^{9} k\,u_k = 1.$$

(The indexing starts at 3 for convenience.) Since there is no unit matrix in the constraint coefficient matrix, we add artificial variables u_1 and u_2, which must vanish at the solution:

$$u_1 + \sum_{k=3}^{9} u_k = 1,$$

$$u_2 + \sum_{k=3}^{9} k u_k = 1.$$

Noting that $|u_k| = \max\{-u_k, u_k\}$, we can transform to the primal problem:

$$\text{Maximize } x_{10}: = x_1 + x_2$$

$$\text{subject to } -1 \le x_i \le 1, \quad i = 3, \ldots, 9, \text{ where}$$

$$\begin{array}{ll} x_3: = x_1 + 3x_2, & x_4: = x_1 + 4x_2 \\ x_5: = x_1 + 5x_2, & x_6: = x_1 + 6x_2 \\ x_7: = x_1 + 7x_2, & x_8: = x_1 + 8x_2 \\ x_9: = x_1 + 9x_2. & \end{array}$$

The variables x_1 and x_2, dual to artificial variables, are not constrained directly.

Input:

```
$IN
   IOBJ = 10,
   M = 8,
   N = 2,
   MIN = .FALSE.,
   INVERT = .TRUE.,
   ITMAX = 1000,
   EPS = 0.,
   ROW = 0, 10, 20, 30, 40, 50, 60, 70,
   COL = 1, 2,
   A(1, 1) = 1., 3.,
   A(1, 2) = 1., 4.,
   A(1, 3) = 1., 5.,
   A(1, 4) = 1., 6.,
```

```
   A(1, 5) = 1., 7.,
   A(1, 6) = 1., 8.,
   A(1, 7) = 1., 9.,
   A(1, 8) = 1., 1.,
   BL = 2*-1.E100, 7*-1., -1.E100,
   BU = 2*1.E100, 7*-1.,  1.E100.
   K = 1,2,3,4,5,6,7,8,9,10,
   X = 2*1.E100,
$END
```

Output:

I	BL(I)	X(I)	BU(I)	T(I, 3)	T(I, 9)
1	-R	2.000	R	1.500	-.500
2	-R	-.333	R	-.167	.167
3	-1.000	1.000	1.000	1.000	-.000
4	-1.000	.667	1.000	.833	.167
5	-1.000	.333	1.000	.667	.333
6	-1.000	.000	1.000	.500	.500
7	-1.000	-.333	1.000	.333	.667
8	-1.000	-.667	1.000	.167	.833
9	-1.000	-1.000	1.000	.000	1.000
10	-R	1.667	R	1.333	-.333

The solution is obtained from row 10: u_3 = 1.333, u_9 = -.333, $u_4, \ldots, u_8 = 0$. The minimal value is 1.667. (Obviously, the exact solution has u_3 = 4/3, u_9 = -1/3.)

```
PROGRAM TEST 73/74      OPT =1        FTN 4.6 + 452                 80/09/25. 13.42.54

      PROGRAM TEST (INPUT , OUTPUT)                                 TEST        2
C                                                                   LINE        2
C------------------------------------------------------------------ LINE        3
C                                                                   LINE        4
C     TEST PROGRAM FOR LINOPT                                       TEST        4
C                                                                   LINE        2
C------------------------------------------------------------------ LINE        3
C                                                                   LINE        4
      DIMENSION A(10,10). ROW (10), COL (10), BL(20), BU(20), K(20) TEST        6
      DIMENSION X(20), U(20), E (100), SCR(10), KORD(10)            TEST        7
      INTEGER ROW, COL                                              TEST        8
C                                                                   TEST        9
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR MOD1       1
      COMMON /XXXLP/ NPI, NPM, IPIV, JPIV, NEGV                     /XXXLP/     3
      COMMON /XXXLP/ BIGM                                           MOD1        2
      LOGICAL MIN, INVERT, NEGV                                     MOD1        3
C                                                                   LINE        2
C------------------------------------------------------------------ LINE        3
C                                                                   LINE

      NAMELIST /IN/                                                 TEST       12
     1   IOBJ, M, N, MIN, INVERT, ITMAX, EPS,                       TEST       13
     2   A, ROW, COL, BL, BU, K, X                                  TEST       14
C                                                                   LINE        2
C------------------------------------------------------------------ LINE        3
C                                                                   LINE        4
        READ IN                                                     TEST       16
        CALL LINOPT(A,ROW,COL,BL,BU,K,X,U,E,SCR)                    TEST       17
        BIGM2 = SQRT(BIGM)                                          TEST       18
        DO 20 I = 1, NPM                                            TEST       19
            IF (BL(I).LE.-BIGM2)  BL(I) = ANEGINF                   TEST       20
            IF(BU(I).GE.BIGM2  BU(I) = -ANEGINF                     TEST       21
            IF (ABS(X(I)).GE.BIGM2 X(I) = SIGN(ANEGINF,X(I))        TEST       22
20          CONTINUE                                                TEST       23
        CALL TABLO (A,ROW,COL,BL,BU,K,X,E,SCR,KORD)                 TEST       24
        PRINT*, 1H                                                  TEST       25
        PRINT*, 10H    ITER = , ITER, 10H    IERR = , IERR          TEST       26
        STOP                                                        TEST       27
        END                                                         TEST       28
```

FIGURE 2 TEST PROGRAM

CHAPTER 6

LISTING

```
      SUBROUTINE LINOPT(A,ROW,COL,BL,BU,K,X,U,E,SCR)
C
C-----------------------------------------------------------------------
C
C     LINEAR PROGRAMMING BY THE DUAL SIMPLEX ALGORITHM
C
C
C     PROBLEM--
C
C        MINIMIZE OR MAXIMIZE  X(IOBJ)  SUBJECT TO
C
C           X(N+I)  =  SUM(J = 1,...,N)  A(ROW(I)+COL(J)) * X(J),
C                                              I = 1,...,M,
C
C           BL(J)  .LE.  X(J)  .LE.  BU(J),  J = 1,...,N+M.
C
C                         ------------
C
C     FURTHER DOCUMENTATION AND EXAMPLES OF USE CAN BE FOUND IN--
C
C        NSWC TR 80-413,  LINOPT, A FORTRAN ROUTINE FOR SOLVING
C        LINEAR PROGRAMMING PROBLEMS, BY J.W. WINGATE.
C
C                         ------------
C
C     ARRAYS ARE PASSED AS FORMAL PARAMETERS, SIMPLE VARIABLES AS
C     ELEMENTS OF THE COMMON BLOCK /XXXLP/.
C
C     INPUTS--THE FOLLOWING VARIABLES AND ARRAYS MUST BE DEFINED ON
C             ENTRY
C
C        IOBJ       INDEX OF THE OBJECTIVE VARIABLE.          (INTEGER)
C                   (NOTE THAT  X(IOBJ)  IS ALSO CONSIDERED AS
C                   A CONSTRAINED VARIABLE.)
C
C        M          NUMBER OF DEPENDENT VARIABLES.            (INTEGER)
C        N          NUMBER OF INDEPENDENT VARIABLES.          (INTEGER)
C
C        MIN        .TRUE. FOR MINIMIZATION,                  (LOGICAL)
C                   .FALSE. FOR MAXIMIZATION.
C
```

```
C         INVERT     .TRUE. IF THE INVERSE MATRIX E            (LOGICAL) LINOPT
C                    IS TO BE CALCULATED.                                LINOPT
C                    .FALSE. IF E IS ALREADY SET TO THE INVERSE FOR      LINOPT
C                    THE BASIS DEFINED BY K.   (FOR REOPTIMIZATION,      LINOPT
C                    INVERT SHOULD BE .FALSE. UNLESS REINVERSION         LINOPT
C                    IS DESIRED.)                                        LINOPT
C                                                                        LINOPT
C         ITMAX      MAXIMUM NUMBER OF ITERATIONS ALLOWED.     (INTEGER) LINOPT
C                    CONTROL IS RETURNED TO THE CALLING                  LINOPT
C                    PROGRAM AFTER ITMAX ITERATIONS.                     LINOPT
C                                                                        LINOPT
C         EPS        ZERO TOLERANCE.   CONSTRAINT VIOLATIONS      (REAL) LINOPT
C                    OR TABLEAU ENTRIES .LE. EPS IN MAGNITUDE            LINOPT
C                    ARE TREATED AS ZERO.                                LINOPT
C                                                                        LINOPT
C         A          ARRAY CONTAINING THE COEFFICIENT MATRIX.     (REAL) LINOPT
C         ROW        ROW INDEX ARRAY.                          (INTEGER) LINOPT
C         COL        COLUMN INDEX ARRAY.                       (INTEGER) LINOPT
C                    THE COEFFICIENT OF  X(J)  IN THE EQUATION           LINOPT
C                    FOR  X(N+I)  IS  A(ROW(I)+COL(J)).                  LINOPT
C                    EITHER (CASE 1)                                     LINOPT
C                       ONE HAS VECTORS  AROW1,...,AROWM  WITH           LINOPT
C                       AROWI(COL(J))  THE COEFFICIENT OF  X(J)          LINOPT
C                       IN THE EQUATION FOR  X(N+I), IN WHICH CASE       LINOPT
C                          ROW(I) = LOCF(AROWI) - LOCF(A),  I = 1,...,M, LINOPT
C                    OR (CASE 2)                                         LINOPT
C                       ONE HAS VECTORS  ACOL1,...,ACOLN  WITH           LINOPT
C                       ACOLJ(ROW(I))  THE COEFFICIENT OF  X(J)          LINOPT
C                       IN THE EQUATION FOR  X(N+I), IN WHICH CASE       LINOPT
C                          COL(J) = LOCF(ACOLJ) - LOCF(A),  J = 1,...,N. LINOPT
C                    (E.G. IF  A  IS DIMENSIONED FOR  MM  ROWS           LINOPT
C                    AND THE COEFFICIENT MATRIX IS STORED IN             LINOPT
C                    THE FIRST  M  ROWS AND  N  COLUMNS OF  A (CASE 2),  LINOPT
C                          ROW(I) = I,  I = 1,...,M,                     LINOPT
C                          COL(J) = (J-1)*MM,  J = 1,...,N,              LINOPT
C                    WHILE IF THE COEFFICIENT MATRIX IS STORED           LINOPT
C                    TRANSPOSED IN THE FIRST  N  ROWS AND  M  COLUMNS    LINOPT
C                    (CASE 1),                                           LINOPT
C                          ROW(I) = (I-1)*MM,  I = 1,...,M,              LINOPT
C                          COL(J) = J,  J = 1,...,N.                     LINOPT
C                    ROW AND COL MAY BE PERMUTED IN ANY CONVENIENT       LINOPT
C                    WAY.)                                               LINOPT
C                                                                        LINOPT
C         BL         ARRAY OF LOWER BOUNDS.                       (REAL) LINOPT
C         BU         ARRAY OF UPPER BOUNDS.                       (REAL) LINOPT
C                                                                        LINOPT
C         K          BASIC SOLUTION KEY.                       (INTEGER) LINOPT
C                    K, IN CONJUNCTION WITH X, SPECIFIES A               LINOPT
C                    PARTICULAR BASIC SOLUTION.  THE EQUATIONS           LINOPT
C                    RELATING  X(N+I), I = 1,...,M  TO                   LINOPT
C                    X(J), J = 1,...,N  (THE CONSTRAINT EQUATIONS)       LINOPT
C                    CAN BE SOLVED FOR VARIOUS COMBINATIONS OF           LINOPT
```

```
C                    M  VARIABLES (SECONDARY VARIABLES) IN TERMS
C                    OF THE REMAINING  N  VARIABLES (PRIMARY
C                    VARIABLES).  K, A PERMUTATION OF  (1,...,N+M),
C                    SPECIFIES SUCH A PARTITION INTO PRIMARY AND
C                    SECONDARY VARIABLES.  K(1),...,K(N)  ARE THE
C                    INDICES OF THE PRIMARY VARIABLES.  K(N+1),...,
C                    K(N+M)  ARE THE INDICES OF THE SECONDARY
C                    VARIABLES.  FOR THE DUAL VARIABLES U(J),
C                    J = 1,...,N+M, PRIMARY AND SECONDARY INDICES
C                    SWITCH ROLES,  U(K(N+1)),...,U(K(N+M)) BEING
C                    PRIMARY.  A BASIC SOLUTION IS SPECIFIED BY
C                    SETTING EACH PRIMAL PRIMARY VARIABLE TO
C                    EITHER OF ITS BOUNDS AND EACH DUAL PRIMARY
C                    VARIABLE TO ZERO.  THE INPUT VALUES OF THE
C                    PRIMAL PRIMARY VARIABLES ARE SWITCHED TO THE
C                    OPPOSITE BOUND IF NECESSARY IN ORDER TO CREATE
C                    A DUAL-FEASIBLE BASIC SOLUTION.
C
C       X            PRIMAL SOLUTION ARRAY.                    (REAL)
C                    X(K(J))  MUST BE SET TO EITHER  BL(K(J)) OR
C                    BU(K(J)), J = 1,...,N. THESE ARE DEFAULT
C                    VALUES TO BE USED WHEN A VANISHING  U(K(J))
C                    MAKES  X(K(J))  INDETERMINATE IN SETTING UP
C                    A DUAL-FEASIBLE SOLUTION.
C
C    OUTPUTS--THE FOLLOWING VARIABLES AND ARRAYS ARE DEFINED
C             OR REDEFINED ON EXIT
C
C       ITER         NUMBER OF ITERATIONS SINCE THE LAST       (INTEGER)
C                    INVERSION.
C
C       IERR         ERROR FLAG.                               (INTEGER)
C                    IERR = 0--OPTIMUM FOUND.
C                           1--INCONSISTENT CONSTRAINTS.
C                           2--ITERATION LIMIT REACHED.
C                           3--INVERSION FAILED (BAD INITIAL BASIS).
C
C       K            BASIC SOLUTION KEY.                       (INTEGER)
C                    SET FOR THE CURRENT BASIS.
C
C       X            PRIMAL SOLUTION ARRAY.                    (REAL)
C
C       U            DUAL SOLUTION ARRAY.                      (REAL)
C                    U(J)  IS THE DUAL VARIABLE (LAGRANGE
C                    MULTIPLIER) FOR THE CONSTRAINTS ON  X(J).
C                    IT IS POSITIVE IF THE UPPER BOUND IS ACTIVE,
C                    NEGATIVE IF THE LOWER BOUND IS ACTIVE.
C
C       E            INVERSE MATRIX ARRAY.                     (REAL)
C                    X(I)  =  SUM (J = 1,...,N)  E(I,J) * X(K(J)),
C                                                   I = 1,...,N.
```

```
C
C       SCR        (ALIAS AROW IN SUBROUTINES) SCRATCH ARRAY.
C
C                      ------------
C
C     MINIMUM DECLARED ARRAY SIZES--
C
C       A          M * N
C       ROW        M
C       COL        N
C       BL         M + N
C       BU         M + N
C       K          M + N
C       X          M + N
C       U          M + N
C       E          M * M
C       SCR        M
C
C                      ------------
C
C     SUBROUTINE TABLO (Q.V.) PRINTS THE FULL EXPLICIT TABLEAU.
C     IT IS NOT CALLED THROUGH LINOPT AND MUST BE CALLED SEPARATELY.
C
C-----------------------------------------------------------------------
C
      DIMENSION BL(1), BU(1), K(1), X(1), U(1), SCR(1)
C
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV
      COMMON /XXXLP/ BIGM
      LOGICAL MIN, INVERT, NEGV
C
C-----------------------------------------------------------------------
C
C     THE VARIABLE  BIGM  REPRESENTS A VERY LARGE NUMBER.  THE DEFAULT
C     VALUE IS 1.E100.  THE USER MAY RESET THIS VALUE IF SO DESIRED.
C     BIGM OR -BIGM MAY BE USED TO FILL IN MISSING UPPER OR LOWER
C     BOUNDS.
C
      DATA BIGM /1.E100/
C
C-----------------------------------------------------------------------
C
      NP1 = N + 1
      NPM = N + M
      IF (.NOT. INVERT)  GO TO 10
         CALL SETINV(A,ROW,COL,K,E,SCR)
         IF (IERR.EQ.3)  RETURN
   10 CONTINUE
      CALL GETROW(A,ROW,COL,E,IOBJ,SCR)
      DO 50 J = 1, N
         KJ = K(J)
```

```
            IF (MIN)  SCR(J) = -SCR(J)
            U(KJ) = SCR(J)
            IF (U(KJ))  20, 40, 30
C           NEGATIVE
   20          CONTINUE
               X(KJ) = BL(KJ)
               GO TO 40
C           POSITIVE
   30          CONTINUE
               X(KJ) = BU(KJ)
   40       CONTINUE
   50       CONTINUE
         DO 60 J = NP1, NPM
            U(K(J)) = 0.
   60       CONTINUE
         CALL PSOL(A,ROW,COL,K,X,E)
         CALL DSIMP(A,ROW,COL,BL,BU,K,X,U,E,SCR)
         IF (IERR.NE.0)  GO TO 110
C           ROUND X-VALUES WITHIN EPS OF BOUNDS
            DO 100 I = 1, NPM
               IF (ABS(X(I)-BL(I)).LE.EPS)  X(I) = BL(I)
               IF (ABS(X(I)-BU(I)).LE.EPS)  X(I) = BU(I)
  100          CONTINUE
  110 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DSIMP(A,ROW,COL,BL,BU,K,X,U,E,AROW)                    DSIMP      1
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
C     DUAL SIMPLEX ALGORITHM                                            DSIMP      4
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1)                 DSIMP      6
C                                                                       DSIMP      7
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR    /XXXLP/    2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                         /XXXLP/    3
      COMMON /XXXLP/ BIGM                                               /XXXLP/    4
      LOGICAL MIN, INVERT, NEGV                                         /XXXLP/    5
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      IERR = 2                                                          DSIMP     10
C     WHEN ITMAX.LT.1 THE LOOP IS PARTIALLY EXECUTED                    DSIMP     11
      DO 100 II = 1, ITMAX                                              DSIMP     12
         CALL PIVROW(BL,BU,K,X)                                         DSIMP     13
         IF (IPIV.GT.0)  GO TO 10                                       DSIMP     14
C           NO PIVOT ROW INDICATES THAT X IS OPTIMAL                    DSIMP     15
            IERR = 0                                                    DSIMP     16
            RETURN                                                      DSIMP     17
   10    CONTINUE                                                       DSIMP     18
         KROW = K(IPIV)                                                 DSIMP     19
         CALL GETROW(A,ROW,COL,E,KROW,AROW)                             DSIMP     20
         CALL PIVCOL(BL,BU,K,X,U,AROW)                                  DSIMP     21
         IF (JPIV.GT.0)  GO TO 40                                       DSIMP     22
C           NO PIVOT COLUMN INDICATES THAT THE CONSTRAINTS              DSIMP     23
C           ARE INCONSISTENT                                            DSIMP     24
            IERR = 1                                                    DSIMP     25
            RETURN                                                      DSIMP     26
   40    CONTINUE                                                       DSIMP     27
         IF (ITMAX.LT.1)  RETURN                                        DSIMP     28
C        NEW SOLUTION KEY                                               DSIMP     29
         K(IPIV) = K(JPIV)                                              DSIMP     30
         K(JPIV) = KROW                                                 DSIMP     31
         CALL NEWINV(E,AROW)                                            DSIMP     32
C        NEW DUAL SOLUTION                                              DSIMP     33
         CALL GETROW(A,ROW,COL,E,IOBJ,AROW)                             DSIMP     34
         DO 70 J = 1, N                                                 DSIMP     35
            IF (MIN)  AROW(J) = -AROW(J)                                DSIMP     36
            U(K(J)) = AROW(J)                                           DSIMP     37
   70       CONTINUE                                                    DSIMP     38
         U(K(IPIV)) = 0.                                                DSIMP     39
C        NEW PRIMAL SOLUTION                                            DSIMP     40
         X(KROW) = BU(KROW)                                             DSIMP     41
         IF (NEGV)  X(KROW) = BL(KROW)                                  DSIMP     42
         CALL PSOL(A,ROW,COL,K,X,E)                                     DSIMP     43
         ITER = ITER + 1                                                DSIMP     44
  100    CONTINUE                                                       DSIMP     45
```

```
                                                            DSIMP     46
      RETURN                                                DSIMP     47
      END
```

```
      SUBROUTINE PIVROW(BL,BU,K,X)
C
C-----------------------------------------------------------------------
C
C     PIVOT ROW SELECTION
C
C-----------------------------------------------------------------------
C
      DIMENSION BL(1), BU(1), K(1), X(1)
C
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV
      COMMON /XXXLP/ BIGM
      LOGICAL MIN, INVERT, NEGV
C
C-----------------------------------------------------------------------
C
      IPIV = 0
      IF (NPM.LT.NP1)  RETURN
      VIOL = 0.
      DO 50 II = NP1, NPM
         I = K(II)
C        CHECK CONSTRAINTS ON X(I)
            D = X(I) - BL(I)
            IF (D.GE.-EPS)  GO TO 10
               D = -D
               IF (VIOL.GT.D)  GO TO 40
               VIOL = D
               IPIV = II
               NEGV = .TRUE.
               GO TO 40
   10       CONTINUE
            D = X(I) - BU(I)
            IF (D.LE.EPS)  GO TO 30
               IF (VIOL.GT.D)  GO TO 40
               VIOL = D
               IPIV = II
               NEGV =.FALSE.
   30       CONTINUE
   40    CONTINUE
   50    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PIVCOL(BL,BU,K,X,U,AROW)                               PIVCOL     2
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
C     PIVOT COLUMN SELECTION                                            PIVCOL     4
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1)                 PIVCOL     6
C                                                                       PIVCOL     7
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR    /XXXLP/    2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                         /XXXLP/    3
      COMMON /XXXLP/ BIGM                                               /XXXLP/    4
      LOGICAL MIN, INVERT, NEGV                                         /XXXLP/    5
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      JPIV = 0                                                          PIVCOL    10
      W = BIGM                                                          PIVCOL    11
      DO 30 JJ = 1, N                                                   PIVCOL    12
         J = K(JJ)                                                      PIVCOL    13
         AA = AROW(JJ)                                                  PIVCOL    14
         IF (NEGV)  AA = -AA                                            PIVCOL    15
         IF (AA.GE.0. .AND. X(J).EQ.BL(J))  GO TO 20                    PIVCOL    16
         IF (AA.LE.0. .AND. X(J).EQ.BU(J))  GO TO 20                    PIVCOL    17
            R = U(J)/AA                                                 PIVCOL    18
            IF (R.GT.W)  GO TO 10                                       PIVCOL    19
            IF (R.EQ.W .AND. RANF(AA).GT.0.5)  GO TO 10                 PIVCOL    20
               JPIV = JJ                                                PIVCOL    21
               W = R                                                    PIVCOL    22
   10       CONTINUE                                                    PIVCOL    23
   20    CONTINUE                                                       PIVCOL    24
   30    CONTINUE                                                       PIVCOL    25
      RETURN                                                            PIVCOL    26
      END                                                               PIVCOL    27
```

```
      SUBROUTINE GETROW(A,ROW,COL,E,KROW,AROW)                          GETROW   1
C                                                                       LINE     2
C---------------------------------------------------------------------- LINE     3
C                                                                       LINE     4
C     GENERATION OF CONSTRAINT COEFFICIENTS FOR THE CURRENT BASIS      GETROW   4
C                                                                       LINE     2
C---------------------------------------------------------------------- LINE     3
C                                                                       LINE     4
      DIMENSION A(1), ROW(1), COL(1), E(1), AROW(1)                     GETROW   6
      INTEGER ROW, COL                                                  GETROW   7
C                                                                       GETROW   8
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR    /XXXLP/  2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                         /XXXLP/  3
      COMMON /XXXLP/ BIGM                                               /XXXLP/  4
      LOGICAL MIN, INVERT, NEGV                                         /XXXLP/  5
C                                                                       LINE     2
C---------------------------------------------------------------------- LINE     3
C                                                                       LINE     4
      IF (KROW.GT.N)  GO TO 20                                          GETROW  11
C        ORIGINAL INDEPENDENT VARIABLE.  GET ROW KROW OF THE INVERSE.   GETROW  12
         JJ = 0                                                         GETROW  13
         DO 10 J = 1, N                                                 GETROW  14
            AROW(J) = E(KROW+JJ)                                        GETROW  15
            IF (ABS(AROW(J)).LE.EPS)  AROW(J) = 0.                      GETROW  16
            JJ = JJ + N                                                 GETROW  17
   10       CONTINUE                                                    GETROW  18
         GO TO 50                                                       GETROW  19
   20 CONTINUE                                                          GETROW  20
C        ORIGINAL DEPENDENT VARIABLE.                                   GETROW  21
C        MULTIPLY ORIGINAL ROW BY THE INVERSE.                          GETROW  22
         KK = ROW(KROW-N)                                               GETROW  23
         JJ = 0                                                         GETROW  24
         DO 40 J = 1, N                                                 GETROW  25
            AROW(J) = 0.                                                GETROW  26
            DO 30 I = 1, N                                              GETROW  27
               AROW(J) = AROW(J) + A(KK+COL(I))*E(I+JJ)                 GETROW  28
   30          CONTINUE                                                 GETROW  29
            IF (ABS(AROW(J)).LE.EPS)  AROW(J) = 0.                      GETROW  30
            JJ = JJ + N                                                 GETROW  31
   40       CONTINUE                                                    GETROW  32
   50 CONTINUE                                                          GETROW  33
      RETURN                                                            GETROW  34
      END                                                               GETROW  35
```

```
      SUBROUTINE PSOL(A,ROW,COL,K,X,E)                                  PSOL       2
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
C     PRIMAL SOLUTION                                                   PSOL       4
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      DIMENSION A(1), ROW(1), COL(1), K(1), X(1), E(1)                  PSOL       6
      INTEGER ROW, COL                                                  PSOL       7
C                                                                       PSOL       8
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR    /XXXLP/    2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                         /XXXLP/    3
      COMMON /XXXLP/ BIGM                                               /XXXLP/    4
      LOGICAL MIN, INVERT, NEGV                                         /XXXLP/    5
C                                                                       LINE       2
C---------------------------------------------------------------------- LINE       3
C                                                                       LINE       4
      DO 30 I = NP1, NPM                                                PSOL      11
         KI = K(I)                                                      PSOL      12
         IF (KI.GT.N)  GO TO 20                                         PSOL      13
            X(KI) = 0.                                                  PSOL      14
            JJ = 0                                                      PSOL      15
            DO 10 J = 1, N                                              PSOL      16
               X(KI) = X(KI) + E(KI+JJ) * X(K(J))                       PSOL      17
               JJ = JJ + N                                              PSOL      18
   10          CONTINUE                                                 PSOL      19
   20    CONTINUE                                                       PSOL      20
   30    CONTINUE                                                       PSOL      21
      DO 60 I = NP1, NPM                                                PSOL      22
         KI = K(I)                                                      PSOL      23
         IF (KI.LE.N)  GO TO 50                                         PSOL      24
            X(KI) = 0.                                                  PSOL      25
            KK = ROW(KI-N)                                              PSOL      26
            DO 40 J = 1, N                                              PSOL      27
               X(KI) = X(KI) + A(KK+COL(J)) * X(J)                      PSOL      28
   40          CONTINUE                                                 PSOL      29
   50    CONTINUE                                                       PSOL      30
   60    CONTINUE                                                       PSOL      31
      RETURN                                                            PSOL      32
      END                                                               PSOL      33
```

```
      SUBROUTINE SETINV(A,ROW,COL,K,E,AROW)
C
C-----------------------------------------------------------------------
C
C     INITIAL INVERSE
C
C-----------------------------------------------------------------------
C
      DIMENSION K(1), E(1), AROW(1)
C
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV
      COMMON /XXXLP/ BIGM
      LOGICAL MIN, INVERT, NEGV
C
C-----------------------------------------------------------------------
C
C     SET E TO THE IDENTITY
      JJ = 0
      DO 20 J = 1, N
         DO 10 I = 1, N
            E(I+JJ) = 0.
   10       CONTINUE
         E(J+JJ) = 1.
         JJ = JJ + N
   20    CONTINUE
C     GENERATE INITIAL INVERSE
      DO 30 J = 1, N
         K(J) = -K(J)
   30    CONTINUE
      DO 90 JJ = 1, N
         DO 40 J = 1, N
            IF (K(J).LT.0)  GO TO 50
   40       CONTINUE
   50    CONTINUE
         KROW = -K(J)
         CALL GETROW(A,ROW,COL,E,KROW,AROW)
         ROWMAX = 0.
         DO 70 L = 1, N
            TEST = ABS(AROW(L))
            IF (K(L).GT.0 .OR. TEST.LT.ROWMAX)  GO TO 60
               ROWMAX = TEST
               JPIV = L
   60       CONTINUE
   70       CONTINUE
         IF (ROWMAX.GT.0.)  GO TO 80
            IERR = 3
            RETURN
   80    CONTINUE
```

```
      K(J) = K(JPIV)                                                    SETINV  42
      K(JPIV) = KROW                                                    SETINV  43
      CALL NEWINV(E,AROW)                                               SETINV  44
   90 CONTINUE                                                          SETINV  45
      ITER = 0                                                          SETINV  46
      RETURN                                                            SETINV  47
      END                                                               SETINV  48
```

```
      SUBROUTINE NEWINV(E,AROW)                                                    NEWINV     2
C                                                                                  LINE       2
C--------------------------------------------------------------------------------- LINE       3
C                                                                                  LINE       4
C     INVERSE UPDATE BY COLUMN OPERATIONS                                          NEWINV     4
C                                                                                  LINE       2
C--------------------------------------------------------------------------------- LINE       3
C                                                                                  LINE       4
      DIMENSION E(1), AROW(1)                                                      NEWINV     6
C                                                                                  NEWINV     7
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR               /XXXLP/    2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                                    /XXXLP/    3
      COMMON /XXXLP/ BIGM                                                          /XXXLP/    4
      LOGICAL MIN, INVERT, NEGV                                                    /XXXLP/    5
C                                                                                  LINE       2
C--------------------------------------------------------------------------------- LINE       3
C                                                                                  LINE       4
      JJPIV = (JPIV-1)*N                                                           NEWINV    10
      DO 20 I = 1, N                                                               NEWINV    11
         EPIV = E(I+JJPIV)/AROW(JPIV)                                              NEWINV    12
         JJ = 0                                                                    NEWINV    13
         DO 10 J = 1, N                                                            NEWINV    14
            E(I+JJ) = E(I+JJ) - EPIV*AROW(J)                                       NEWINV    15
            JJ = JJ + N                                                            NEWINV    16
   10       CONTINUE                                                               NEWINV    17
         E(I+JJPIV) = EPIV                                                         NEWINV    18
   20    CONTINUE                                                                  NEWINV    19
      RETURN                                                                       NEWINV    20
      END                                                                          NEWINV    21
```

```
      SUBROUTINE TABLO(A,ROW,COL,BL,BU,K,X,E,SCR,KORD)                  TABLO      2
C                                                                       LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                       LINE       4
C     TABLEAU PRINTOUT                                                  TABLO      4
C                                                                       TABLO      5
C     KORD IS AN ARRAY OF LENGTH AT LEAST N USED FOR REORDERING         TABLO      6
C     K(1),...,K(N) IN ASCENDING ORDER.                                 TABLO      7
C     SEE LINOPT FOR DESCRIPTIONS OF THE OTHER PARAMETERS.              TABLO      8
C                                                                       TABLO      9
C     LINOPT MUST HAVE BEEN CALLED BEFORE CALLING TABLO.                TABLO     10
C                                                                       LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                       LINE       4
      DIMENSION A(1), ROW(1), COL(1), BL(1), BU(1), K(1), X(1), E(1)    TABLO     12
      DIMENSION SCR(1), KORD(1)                                         TABLO     13
      INTEGER ROW, COL                                                  TABLO     14
C                                                                       TABLO     15
      COMMON /XXXLP/ IOBJ, M, N, MIN, INVERT, ITMAX, EPS, ITER, IERR    /XXXLP/    2
      COMMON /XXXLP/ NP1, NPM, IPIV, JPIV, NEGV                         /XXXLP/    3
      COMMON /XXXLP/ BIGM                                               /XXXLP/    4
      LOGICAL MIN, INVERT, NEGV                                         /XXXLP/    5
C                                                                       LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                       LINE       4
    1 FORMAT (1H1//T55,*T A B L E A U*//1X,*  I *,5X,*BL(I)*,           TABLO     18
     /        6X,*X(I)*,5X,*BU(I)*,1X,10A10/(36X,10A10))                TABLO     19
    2 FORMAT (1H0,I3,1X,3F10.3,1X,10F10.3/(36X,10F10.3))                TABLO     20
    3 FORMAT (2X,*T(I,*,I3,*)*)                                         TABLO     21
    4 FORMAT (///1H0,*THE OBJECTIVE VARIABLE, X(*,I3,*), IS TO BE *,A10) TABLO     22
C                                                                       LINE       2
C-----------------------------------------------------------------------LINE       3
C                                                                       LINE       4
      DO 110 J = 1, N                                                   TABLO     24
         KORD(J) = J                                                    TABLO     25
  110    CONTINUE                                                       TABLO     26
      DO 130 J = 1, N                                                   TABLO     27
         JMIN = J                                                       TABLO     29
         DO 120 JJ = J, N                                               TABLO     29
            IF (K(KORD(JJ)).LT.K(KORD(JMIN)))  JMIN = JJ                TABLO     30
  120       CONTINUE                                                    TABLO     31
         KTEMP = KORD(J)                                                TABLO     32
         KORD(J) = KORD(JMIN)                                           TABLO     33
         KORD(JMIN) = KTEMP                                             TABLO     34
  130    CONTINUE                                                       TABLO     35
      DO 10 J = 1, N                                                    TABLO     36
         ENCODE (10,3,SCR(J)) K(KORD(J))                                TABLO     37
   10    CONTINUE                                                       TABLO     38
      PRINT 1, (SCR(J), J = 1, N)                                       TABLO     39
```

```
      DO 20 I = 1, NPM                                                  TABLO
         CALL GETROW(A,ROW,COL,E,I,SCR)                                 TABLO
         PRINT 2, I, BL(I), X(I), BU(I), (SCR(KORD(J)), J = 1, N)       TABLO
   20    CONTINUE                                                       TABLO
      OPT = 10HMAXIMIZED.                                               TABLO
      IF (MIN)  OPT = 10HMINIMIZED.                                     TABLO
      PRINT 4, IOBJ, OPT                                                TABLO
      RETURN                                                            TABLO
      END                                                               TABLO
```

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